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The $N(p, 2\pi)2N$ reactions near threshold

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Abstract. A preliminary calculation of the total cross sections for the reactions $pn \to pn\pi^+\pi^-$, $pp \to pp\pi^+\pi^-$, $pp \to pp\pi^0\pi^0$, and $pp \to nn\pi^+\pi^+$ was carried out for the incident proton energy range $E_p \lesssim 850$ MeV. This calculation was done in the framework of a model based on a version of the standard effective chiral Lagrangian, for which only three- and four-pion diagrams were taken into account together with diagrams containing two two-pion vertices. The invariant amplitudes were calculated to the threshold approximation. It was shown that for a reliable description of the reactions under consideration it is necessary to take into account the $N^*(1440)$ mechanism too. The final state interaction of the nucleons was also considered. Results of the calculation are compared with available experimental data.

PACS. 13.75.-n Hadron-induced low- and intermediate-energy reactions and scattering (energy ≤ 10 GeV) – 13.75.Cs Nucleon-nucleon interactions

1 Introduction

Systematic investigations of the processes involving the exitation of multipion states would provide a natural test for nonlinear pion-nucleon dynamics generated by the effective chiral Lagrangians. Such investigations can make clear to what extent those Lagrangians describe processes of production and absorption of pions in hadron-hadron and hadron-nucleus collisions. A particular aim of such investigations can be estimation of the chiral symmetry breaking parameter ξ. From that point of view experimental and theoretical studies of the reactions $(\pi, 2\pi)$ and $(p, 2\pi)$ on the nucleons and nuclei, especially close to threshold, are very important. But while the $(\pi, 2\pi)$ reactions on nucleons are studied (and being studied) relatively well (see e.g. [1–3]), reactions $(p, 2\pi)$ near threshold are practically unstudied. Experimental data on the $N(p, 2\pi)2N$ [4–6] are available for incident proton momenta $p_p \geq 1.379\,\mathrm{GeV/c}$ ($E_p \geq 0.730$ GeV), while the threshold momentum value for the $pp \to pp\pi^+\pi^-$ reaction is $p_p^{th} = 1.219\,\mathrm{GeV/c}$ ($E_p^{th} = 0.600$ GeV). Note that $(p, 2\pi)$ reactions present more "pure" examples of double pion production than the $(\pi, 2\pi)$ reactions which already involve one pion in the initial state and, to some extent, can be regarded as a kind of "quasielastic scattering of pions".

Studying the $(p, 2\pi)$ reactions on nuclei has some additional aspects of interest. The resonance-like behaviour of inclusive pion production by protons on Cu and Ga was observed in the experiments [7–9]. Such a behaviour was seen in the low-energy part (< 70 MeV) of the pion spectra at the incident proton energy of 350 MeV and was interpreted as double pion production showing a narrow resonance structure: the width of that 350 MeV resonance was estimated to be of about 7 MeV [9]. This effect needs to be confirmed (or refuted) experimentally and understood theoretically.

Another interesting aspect of studying the $(p, 2\pi)$ on nuclei is a possibility of the two-pion state formation in nuclear matter [10]. The $(p, 2\pi)$ together with $(\pi, 2\pi)$ can be a suitable tool for looking for such states.

Also, it can be mentioned that a detailed study of the $(p, \pi^-\pi^-)$ reaction on nuclei could shed light on the relative importance of the three-nucleon interaction in nuclei.

All that shows that thorough studies of the $(p, 2\pi)$ reactions near their thresholds are worthwile to be undertaken.

Section 2 of this paper is devoted to calculation of the cross sections for the reactions $pn \to pn\pi^+\pi^-$, $pp \to pp\pi^+\pi^-$, $pp \to pp\pi^0\pi^0$, and $pp \to nn\pi^+\pi^+$ on the base of the effective chiral Lagrangian [11,12]. In Section 3 we show that the $N^*(1440)$ mechanism is very important giving the main contribution to the $pn \to pn\pi^+\pi^-$, $pp \to pp\pi^+\pi^-$, and $pp \to pp\pi^0\pi^0$ reaction amplitudes near threshold. In Section 4 we analyse the two-nucleon final state interaction for the reactions under consideration. Sections 5 and 6 present the numerical results and final remarks.

2 Effective chiral Lagrangian

The standard effective chiral Lagrangian used here to describe the reactions $NN \to NN\pi\pi$ has the folowing form [11,12]

$$\mathcal{L}_{int} = \mathcal{L}_{NN\pi} + \mathcal{L}_{NN\pi\pi\pi} + \mathcal{L}_{NN\pi\pi} + \mathcal{L}_{\pi\pi\pi\pi}, \tag{1}$$

$$\mathcal{L}_{NN\pi} = \frac{g}{2m}\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi(\partial^\mu\boldsymbol{\varphi}), \tag{2}$$

$$\mathcal{L}_{NN\pi\pi\pi} = -\frac{g}{2m}\frac{1}{4f_\pi^2}\bar{\psi}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi(\partial^\mu\boldsymbol{\varphi})(\boldsymbol{\varphi})^2, \tag{3}$$

$$\mathcal{L}_{NN\pi\pi} = -\frac{1}{4f_\pi^2}\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi[\boldsymbol{\varphi}\times\partial^\mu\boldsymbol{\varphi}], \tag{4}$$

$$\mathcal{L}_{\pi\pi\pi\pi} = -\frac{1}{4f_\pi^2}(\boldsymbol{\varphi})^2(\partial^\mu\boldsymbol{\varphi})^2 + \frac{1}{4f_\pi^2}\frac{1}{2}\left(1-\frac{1}{2}\xi\right)m_\pi^2((\boldsymbol{\varphi})^2)^2, \tag{5}$$

where ψ and $\boldsymbol{\varphi}$ are nucleon and pion fields respectively, $g = 13.4$ is the pion-nucleon interaction constant, $f_\pi = 93.2$ MeV is the pion decay constant, ξ is the chiral symmetry breaking parameter introduced in [12] to measure the deviation of the $\pi\pi$ amplitude from the σ model calculation, m and m_π denote the nucleon and pion masses.

It is known [1–3] that the dominant Feynman diagrams for the $(\pi,2\pi)$ reactions not far from threshold are three- and four-meson ones (Fig. 1). Related diagrams (a and b of Fig. 2) are expected to be dominant for the $(p,2\pi)$ reactions too. But it turns out that the contribution to the reaction amplitude of the diagrams c of Fig. 2 (which contain two two-meson vertices) is of the same order of magnitude as the contribution of the diagrams a and b. Hence, one should take into account all the diagrams of Fig. 2 even for the rough estimation of the $(p,2\pi)$ reaction cross sections.

Since we are interested in the $(p,2\pi)$ cross sections not far from threshold, for the preliminary estimation we can use the "threshold approximation" i.e. to calculate the reaction amplitude at threshold.

The total cross section of any $N(p,2\pi)2N$ reaction can be written as

$$\sigma = R(s)\frac{1}{4}\sum_{\mu,\nu,\mu',\nu'}|T_{\mu\nu\to\mu'\nu'}|^2 S\eta, \tag{6}$$

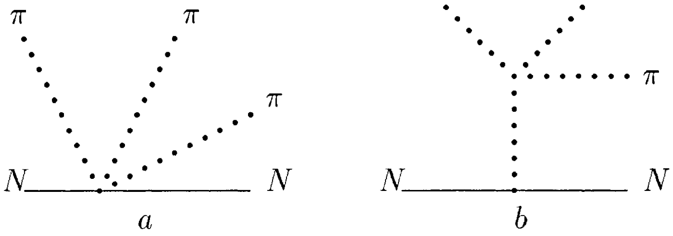

Fig. 1. Dominant diagram types for the $N(\pi,2\pi)N$ reactions (effective chiral Lagrangian): a — three-pion, b — four-pion diagrams

where η is a factor arising on account of two-nucleon final state interaction (see Section IV); S denote the statistical factor connected with numbers of identical particles in the final state, $R(s)$ is the kinematical factor (including the phase space) [13]:

$$R(s) = \frac{m^4 m_\pi(\sqrt{s}-2m-2m_\pi)^{7/2}}{840\pi^4(m+m_\pi)^{3/2}|\boldsymbol{p}|} \tag{7}$$

with $s = (E+2m)^2 - \boldsymbol{p}^2 = 2m(E+2m)$, E and $\boldsymbol{p}$ being the kinetic energy and momentum of the incident proton in the lab. frame. The reaction amplitude $T_{\mu\nu\to\mu'\nu'}$ describes a transition from the state with z-components μ and ν of the bombarding proton spin and the nucleon-target spin to the state in which z-components of the final nucleon spins are equal to μ' and ν'; it is assumed that T is symmetrized (or antisymmetrized) in a appropriate way. The threshold amplitudes $T_{\mu\nu\to\mu'\nu'}$ are presented below for the reactions under consideration, namely for $pn \to pn\pi^+\pi^-$, $pp \to pp\pi^+\pi^-$, $pp \to pp\pi^0\pi^0$, and $pp \to nn\pi^+\pi^+$:

$$\begin{aligned} &T_{\mu\nu\to\mu'\nu'}(pn\to pn\pi^+\pi^-) \\ &= \left[-\left(\frac{g}{2m}\right)^2\frac{1}{4f_\pi^2}4\mu\nu\frac{12mm_\pi\left(1-\frac{5}{6}\xi\right)-8m_\pi^2}{(2m+m_\pi)^2}\right. \\ &\left.+\left(\frac{1}{4f_\pi^2}\right)^2\frac{2m_\pi(2m+m_\pi)}{m(m+m_\pi)}\right](\delta_{\mu'\mu}\delta_{\nu'\nu}-\delta_{\nu'\mu}\delta_{\mu'\nu}) \\ &+\left(\frac{g}{2m}\right)^2\frac{1}{4f_\pi^2}4\mu\nu\frac{4mm_\pi\left(1-\frac{3}{2}\xi\right)}{(2m+m_\pi)^2} \\ &\times(\delta_{\mu'\mu}\delta_{\nu'\nu}+\delta_{\nu'\mu}\delta_{\mu'\nu}); \end{aligned} \tag{8a}$$

$$\begin{aligned} &T_{\mu\nu\to\mu'\nu'}(pp\to pp\pi^+\pi^-) \\ &= \left[\left(\frac{g}{2m}\right)^2\frac{1}{4f_\pi^2}4\mu\nu\frac{8mm_\pi\left(1-\frac{1}{2}\xi\right)-8m_\pi^2}{(2m+m_\pi)^2}\right. \\ &\left.-\left(\frac{1}{4f_\pi^2}\right)^2\frac{2m_\pi(2m+m_\pi)}{m(m+m_\pi)}\right](\delta_{\mu'\mu}\delta_{\nu'\nu}-\delta_{\nu'\mu}\delta_{\mu'\nu}); \end{aligned} \tag{8b}$$

$$\begin{aligned} &T_{\mu\nu\to\mu'\nu'}(pp\to pp\pi^0\pi^0) \\ &= \left(\frac{g}{2m}\right)^2\frac{1}{4f_\pi^2}4\mu\nu\frac{8mm_\pi\left(1-\frac{3}{2}\xi\right)}{(2m+m_\pi)^2} \\ &\times(\delta_{\mu'\mu}\delta_{\nu'\nu}-\delta_{\nu'\mu}\delta_{\mu'\nu}); \end{aligned} \tag{8c}$$

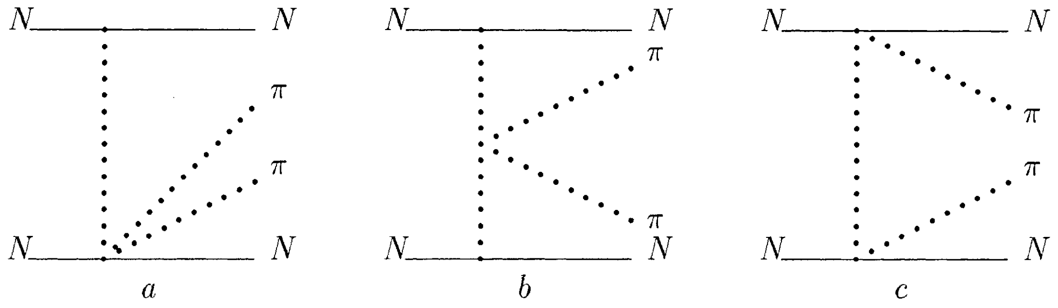

Fig. 2. Dominant diagram types for the $N(p,2\pi)2N$ reactions (effective chiral Lagrangian)

$$
\begin{aligned}
&T_{\mu\nu\to\mu'\nu'}(pp\to nn\pi^+\pi^+)\\
&=\left[-\left(\frac{g}{2m}\right)^2\frac{1}{4f_\pi^2}4\mu\nu\frac{16m_\pi(m\xi-m_\pi)}{(2m+m_\pi)^2}\right.\\
&\left.+\left(\frac{1}{4f_\pi^2}\right)^2\frac{4m_\pi(2m+m_\pi)}{m(m+m_\pi)}\right](\delta_{\mu'\mu}\delta_{\nu'\nu}-\delta_{\nu'\mu}\delta_{\mu'\nu}).
\end{aligned}
\tag{8d}
$$

The statistical factor S is equal to 1 and 0.5 for the reactions (8a) and (8b) respectively, and 0.25 for (8c) and (8d).

Note that contributions T_a and T_b of diagrams a and b to the reaction amplitudes at threshold cancel each other to a large extent. Namely, if one consider these contributions in terms of expansions in powers of the ratio m_π/m, the principal terms of these expansions (which are of the order of $(g/2m)^2(4f_\pi^2)^{-1}(m_\pi/m)^0$) cancel each other, so that T_a+T_b is of the order of $(g/2m)^2(4f_\pi^2)^{-1}(m_\pi/m)$. At the same time $T_c\sim(4f_\pi^2)^{-2}(m_\pi/m)$, i.e. T_c and T_a+T_b are of the same order of magnitude. For example, the contributions of different diagrams to the threshold amplitude of the reaction $pp\to pp\pi^+\pi^-$ are equal to

$$
\begin{aligned}
T_a=&-\left(\frac{g}{2m}\right)^2\frac{1}{4f_\pi^2}4\mu\nu\frac{8(m+m_\pi)}{2m+m_\pi}\\
&\times(\delta_{\mu'\mu}\delta_{\nu'\nu}-\delta_{\nu'\mu}\delta_{\mu'\nu}),
\end{aligned}
\tag{9a}
$$

$$
\begin{aligned}
T_b=&\left(\frac{g}{2m}\right)^2\frac{1}{4f_\pi^2}4\mu\nu\frac{8m\left[2m+\left(4-\frac{1}{2}\xi\right)m_\pi\right]}{(2m+m_\pi)^2}\\
&\times(\delta_{\mu'\mu}\delta_{\nu'\nu}-\delta_{\nu'\mu}\delta_{\mu'\nu}),
\end{aligned}
\tag{9b}
$$

$$
\begin{aligned}
T_a+T_b=&\left(\frac{g}{2m}\right)^2\frac{1}{4f_\pi^2}4\mu\nu\frac{8mm_\pi\left(1-\frac{1}{2}\xi\right)-8m_\pi^2}{(2m+m_\pi)^2}\\
&\times(\delta_{\mu'\mu}\delta_{\nu'\nu}-\delta_{\nu'\mu}\delta_{\mu'\nu}),
\end{aligned}
\tag{9c}
$$

$$
\begin{aligned}
T_c=&-\left(\frac{1}{4f_\pi^2}\right)^2\frac{2m_\pi(2m+m_\pi)}{m(m+m_\pi)}\\
&\times(\delta_{\mu'\mu}\delta_{\nu'\nu}-\delta_{\nu'\mu}\delta_{\mu'\nu}).
\end{aligned}
\tag{9d}
$$

Similar cancellation takes place for other isospin channels of the $(p,2\pi)$ reaction.

3 The $N^*(1440)$ mechanism

It has been shown (see e.g. [14]) that the contribution of the $N^*(1440)$ resonance to $N(\pi,2\pi)N$ reaction amplitudes near threshold can be considerable for the isospin channels in which formation of this resonance in the intermediate state is allowed by the isospin conservation law. It seems that similar situation takes place for $(p,2\pi)$ reactions too. Such a mechanism may be important because near threshold the intermediate $N^*(1440)$ can decay emitting two pions with total angular momentum equal to zero (while one-pion decay allows only emission of p-pions). To describe this mechanism (see Fig. 3) one has to add to Lagrangian (1) two terms corresponding to formation and decay of the $N^*(1440)$ resonance, namely

$$
\mathcal{L}'_{int}=C_1\overline{N^*}\gamma_\mu\gamma_5\boldsymbol{\tau}\psi(\partial^\mu\boldsymbol{\varphi})+C_2\overline{\psi}N^*\boldsymbol{\varphi}^2+h.c.\,, \tag{10}
$$

N^* being the $N^*(1440)$ field, $C_1=g/4m$ [15], $C_2=3.04/m_\pi$ [14].

The threshold amplitude calculated using the interaction Lagrangian $\mathcal{L}_{int}+\mathcal{L}'_{int}$ can be presented in the following form

$$
\begin{aligned}
&T(pn\to pn\pi^+\pi^-)\\
&\quad=\mathcal{P}_0[(3A+a+b)(\boldsymbol{\sigma_1}\hat{\mathbf{p}}_0)(\boldsymbol{\sigma_2}\hat{\mathbf{p}}_0)-2c]\\
&\quad+\mathcal{P}_1(A-a+b)(\boldsymbol{\sigma_1}\hat{\mathbf{p}}_0)(\boldsymbol{\sigma_2}\hat{\mathbf{p}}_0)\,,
\end{aligned}
\tag{11a}
$$

$$
T(pp\to pp\pi^+\pi^-)=2\mathcal{P}_0[(A+a)(\boldsymbol{\sigma_1}\hat{\mathbf{p}}_0)(\boldsymbol{\sigma_2}\hat{\mathbf{p}}_0)-c], \tag{11b}
$$

$$
T(pp\to pp\pi^0\pi^0)=2\mathcal{P}_0(A+d)(\boldsymbol{\sigma_1}\hat{\mathbf{p}}_0)(\boldsymbol{\sigma_2}\hat{\mathbf{p}}_0)\,, \tag{11c}
$$

$$
T(pp\to nn\pi^+\pi^+)=-2\mathcal{P}_0[e(\boldsymbol{\sigma_1}\hat{\mathbf{p}}_0)(\boldsymbol{\sigma_2}\hat{\mathbf{p}}_0)-2c]\,. \tag{11d}
$$

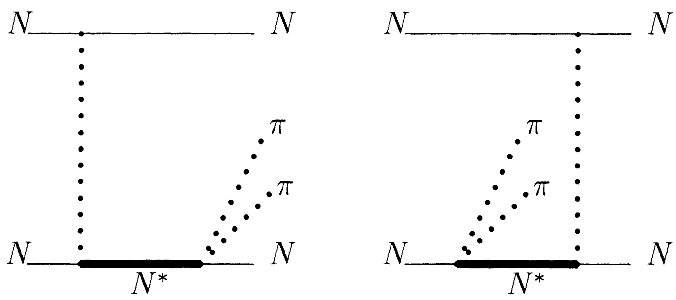

Fig. 3. Diagrams of the $N^*(1440)$ mechanism for $N(p,2\pi)2N$ reactions

Here $\hat{\mathbf{p}}_{\mathbf{0}} = \mathbf{p}/p$ is a unit vector of the incident proton momentum direction, σ's are Pauli matrices; $\mathcal{P}_0 = (1 - \boldsymbol{\sigma_1\sigma_2})/4$ and $\mathcal{P}_1 = (3 + \boldsymbol{\sigma_1\sigma_2})/4$ are projection operators on the singlet and triplet states of a two-nucleon system respectively;

$$A = C_2 \left(\frac{g}{2m}\right)^2 \frac{4}{2m+m_\pi} \times \left[\frac{m+m_\pi}{M-m-2m_\pi} + \frac{(m+m_\pi)(M+m)-2mm_\pi}{M^2-(m^2-4mm_\pi)}\right], \tag{12}$$

M being the $N^*(1440)$ mass ($M = 1,44$ GeV);

$$a = \left(\frac{g}{2m}\right)^2 \frac{1}{4f_\pi^2} \frac{8mm_\pi\left(1-\frac{1}{2}\xi\right) - 8m_\pi^2}{(2m+m_\pi)^2}; \tag{13a}$$

$$b = \left(\frac{g}{2m}\right)^2 \frac{1}{4f_\pi^2} \frac{16mm_\pi(1-\xi) - 8m_\pi^2}{(2m+m_\pi)^2}; \tag{13b}$$

$$c = \left(\frac{1}{4f_\pi^2}\right)^2 \frac{2m_\pi(2m+m_\pi)}{m(m+m_\pi)}; \tag{13c}$$

$$d = \left(\frac{g}{2m}\right)^2 \frac{1}{4f_\pi^2} \frac{8mm_\pi\left(1-\frac{3}{2}\xi\right)}{(2m+m_\pi)^2}; \tag{13d}$$

$$e = \left(\frac{g}{2m}\right)^2 \frac{1}{4f_\pi^2} \frac{16m_\pi(m\xi - m_\pi)}{(2m+m_\pi)^2}. \tag{13e}$$

The main contributions to the amplitudes (11a), (11b), and (11c) are due to the $N^*(1440)$ mechanism: the value of A is equal to 11504 GeV^{-4} (all the values presented here and below are calculated using form factors in the vertices describing emission or absorbtion of a virtual pion — see Section 5); at the same time $c = 384$ GeV^{-4}; values of a, b, and d depend on ξ: for ξ changing between 1 and -1

$$111\,\mathrm{GeV}^{-4} < a < 427\,\mathrm{GeV}^{-4},$$

$$-46\,\mathrm{GeV}^{-4} < b < 1215\,\mathrm{GeV}^{-4},$$

$$-158\,\mathrm{GeV}^{-4} < d < 788\,\mathrm{GeV}^{-4}$$

(for $\xi = 0$ $a = 269\,\mathrm{GeV}^{-4}$, $b = 536\,\mathrm{GeV}^{-4}$, and $d = 315\,\mathrm{GeV}^{-4}$). Thus, the sensitivity of these three amplitudes to the parameter ξ is too low to extract its value from experimental data on the relevant cross sections.

The situation with $T(pp \to nn\pi^+\pi^+)$ is quite different. In this channel excitation of the $N^*(1440)$ is forbidden by the isospin conservation law, and the amplitude does not depend on A as it seen from (11d). For $1 > \xi > -1$ parameter e changes its value between +542 and -732 GeV^{-4}. Therefore the amplitude of this channel and its cross section are very sensitive to ξ. But very low values of this cross section (see below) make the perspective of its measurement doubtful.

4 Two-nucleon final state interaction

The relative momemtum of two final nucleons is rather low near threshold. In this case the interaction between them may essentially modify their spectra and, consequently, values of the cross sections. Taking into account this final state interaction (FSI) the transition amplitude to the state with the relative momentum of two final nucleons $\mathbf{p}$ can be written as follows

$$\tilde{T}(\mathbf{p}) = \int d^3p'\psi_{\mathbf{p}}(\mathbf{p}')T(\mathbf{p}') \approx (2\pi)^{3/2}\varphi^*_{\mathbf{p}}(0)T(0). \tag{14}$$

Here T($\mathbf{p}$) is a reaction amplitude calculated without FSI, $\varphi_{\mathbf{p}}(\mathbf{x})$ and $\psi_{\mathbf{p}}(\mathbf{p}')$ are the wave function of the relative motion of two final nucleons and its Fourier transform, i.e.

$$\varphi_{\mathbf{p}}(\mathbf{x}) = (2\pi)^{-3/2}\int d^3p'\, e^{i\mathbf{p}'\mathbf{x}}\psi_{\mathbf{p}}(\mathbf{p}')$$

(spin indices are omitted here).

For the energy region close to threshold we can consider only S-wave interaction between two final nucleons, i.e. ${}^1S_0$ potential for pp and nn and ${}^1S_0$ and ${}^3S_1$ potentials for pn. In such an approach the pn wave function reads as

$$\varphi_{\mathbf{p}}(\mathbf{x}) = \varphi^s_p(x)\chi_s + \varphi^t_p(x)\chi_t, \tag{15}$$

where χ_s and χ_t are spin functions of the singlet and triplet two-nucleon states, $\varphi^s_p(x)$ and $\varphi^t_p(x)$ are corresponding radial S-wave functions, which we choose as the solutions of the Schrödinger equation with the square-well potentials. The depths and the radii of these potentials are equal to [18]: $V_s = 13.40$ MeV, $V_t = 31.28$ MeV, $R_s = 2.65$ fm, $R_t = 2.205$ fm. As a result

$$\begin{aligned} &(2\pi)^{3/2}[\varphi^j_p(0)]^* = \exp(i\delta_j)\frac{k}{\sqrt{p^2+mV_j\cos^2 kR_j}}, \\ &k^2 = p^2 + mV_j, \end{aligned} \tag{16}$$

where $j = s, t$.

The Coulomb interaction of two final protons was taken into account multiplying the wave function φ^s_p by the Coulomb penetration factor C_0 (see e.g. [19]).

As it was expected the two-nucleon FSI leads to a significant change of final nucleon spectra for incident proton energies close to threshold as compared to spectra given by the phase space. Figure 4 shows two examples of such a change for the $pp \to pp\pi^+\pi^-$ reaction. The factors of increase of the cross sections due to FSI in our approach can be expressed as follows

$$\eta_j = \frac{105}{8(mT_0)^{7/2}}\int_0^{p_{max}} (2\pi)^3|\varphi^j_p(0)|^2(mT_0 - p^2)^2p^2dp, \tag{17}$$

where $T_0 = \sqrt{s} - 2m - 2m_\pi$ is the kinetic energy of the reaction products in the c.m. frame. For example, for $E_p = 0.605$ GeV ($p_p = 1.225$ GeV/c) $\eta_s = 30.2$ (without Coulomb interaction), $\eta^{Coul}_s = 14.1$ (with Coulomb interaction), and $\eta_t = 6.92$ (without Coulomb interaction).

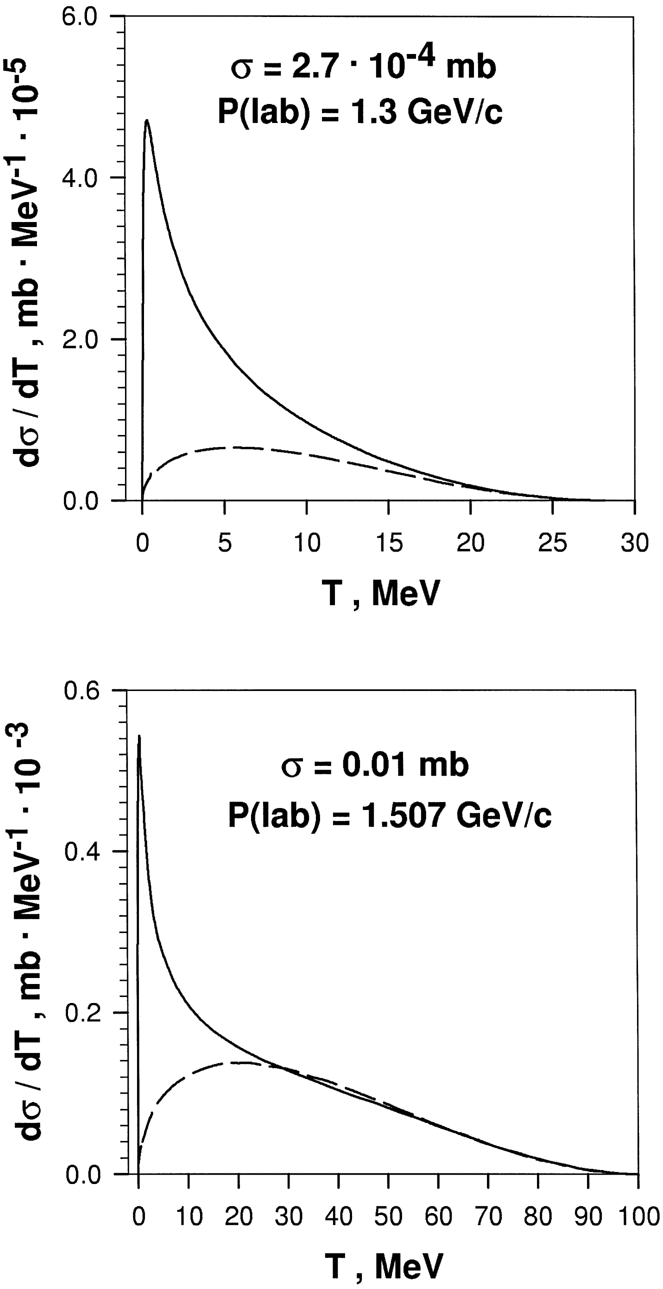

Fig. 4. Final proton spectra calculated for $pp \to pp\pi^+\pi^-$ at the initial proton momenta 1.300 and 1.507 GeV/c: solid line – FSI, dashed line – phase space, T – final proton kinetic energy in the c.m. frame

For $E_p = 0.665$ GeV ($p_p = 1.300$ GeV/c) these factors are equal to 3.66, 2.76, and 2.61 respectively.

Note that taking into account the two-nucleon FSI in $N(N,2\pi)2N$, generally speaking, requires considering the "single-nucleon" mechanism of the two-pion production (the diagram of Fig. 5). Such a mechanism can be described adding a new term to the interaction Lagrangian, namely [1,20]

$$\mathcal{L}'_{NN\pi\pi} = -4\pi(\lambda/m_\pi)\overline{\psi}\boldsymbol{\varphi}^2\psi \tag{18}$$

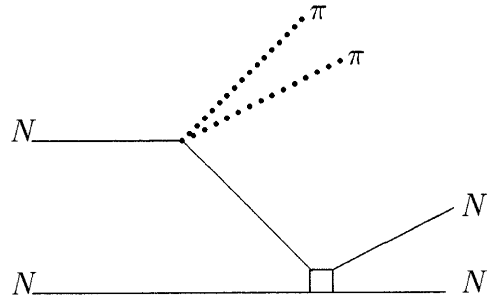

Fig. 5. "Single-nucleon" mechanism diagram of two-pion production

with $\lambda = 0.0075$. However, the calculation shows that the contribution of this mechanism to the threshold amplitude is neglegible.

5 Numerical results

Numerical calculation of cross sections for four reaction channels under consideration were performed by use of formulae (6,7,11–13,17). Since the absolute value of four-momentum Q^2 transfered by off-shell pions is rather high the monopole form factor $F(Q^2)$ was included into corresponding vertices:

$$F(Q^2) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 - Q^2} \tag{19}$$

with the cut-off parameter $\Lambda = 1.75\,\text{GeV}$ [21].

Results are presented in Tables 1–4. Unfortunately, there are no experimental data on $N(p,2\pi)2N$ near threshold to compare our results with. But to get some idea about reliability of the model used here we calculated the cross sections in the energy region where this model cannot be expected to describe experimental data. Nevertheless Tables 1 and 2 show that the model of this paper gives at least the same order of magnitude for calculated cross sections of $pn \to pn\pi^+\pi^-$ and $pp \to pp\pi^+\pi^-$ as measured cross sections have. As it is seen from Tables 1 and 3 the sensitivity of the $pn \to pn\pi^+\pi^-$ and $pp \to pp\pi^0\pi^0$ cross sections to the parameter ξ is rather weak. For $pp \to pp\pi^+\pi^-$ this sensitivity is even lower, and in Table 2 we present its cross section just for $\xi = 0$ because $\sigma(\xi = \pm1)$ differ from $\sigma(\xi = 0)$ only by $\mp1\%$.

As it was mentioned above the $pp \to nn\pi^+\pi^+$ reaction differs significantly from three others (see Table 4). Since the $N^*(1440)$ mechanism is forbidden for it, the cross section is 2–3 orders of magnitude less than for three other reactions. Table 4 shows that this reaction is highly sensitive to ξ. But it should be beared in mind that we can expect this model to be reliable only for energies close to threshold (say, for $E_p < 0.75$ GeV). However, at such energies the cross section values are very small, and their measurement presents a difficult experimental problem. To describe this reaction at higher energies one should refuse the threshold approximation, take into account diagrams of the Lagrangian (1) neglected here (i.e. so-called

Table 1. $\sigma(pn \to pn\pi^+\pi^-)$, mb

		ξ			
p_p, GeV/c	E_p, GeV	1.0	0	-1.0	exp.
1.225	0.605	$1.6 \cdot 10^{-6}$	$1.7 \cdot 10^{-6}$	$1.8 \cdot 10^{-6}$	
1.250	0.625	$1.2 \cdot 10^{-4}$	$1.3 \cdot 10^{-4}$	$1.4 \cdot 10^{-4}$	
1.300	0.665	0.0017	0.0018	0.0019	
1.379	0.730	0.011	0.012	0.013	0.020 ± 0.007 [5]
1.424	0.767	0.023	0.023	0.025	
1.437	0.778	0.027	0.028	0.030	0.057 ± 0.013 [5]
1.507	0.837	0.060	0.064	0.068	0.110 ± 0.021 [5]
1.531	0.857	0.077	0.081	0.085	
1.562	0.884	0.102	0.107	0.113	0.208 ± 0.022 [5]

Table 2. $\sigma(pp \to pp\pi^+\pi^-)$, mb

p_p, GeV/c	E_p, Gev	theor.($\xi = 0$)	exp.
1.225	0.605	$1.8 \cdot 10^{-7}$	
1.250	0.625	$1.9 \cdot 10^{-5}$	
1.300	0.665	$2.7 \cdot 10^{-4}$	
1.379	0.730	0.0018	0.008 ± 0.005 [5]
1.424	0.767	0.0036	0.01 ± 0.01 [4]
1.437	0.778	0.0044	0.019 ± 0.07 [5]
1.507	0.837	0.010	0.029 ± 0.011 [5]
1.531	0.857	0.013	0.05 ± 0.02 [4]
1.562	0.884	0.017	0.057 ± 0.010 [5]

Table 3. $\sigma(pp \to pp\pi^0\pi^0)$, mb

		ξ		
p_p, GeV/c	E_p, GeV	1.0	0	-1.0
1.225	0.605	$2.6 \cdot 10^{-5}$	$2.8 \cdot 10^{-5}$	$3.1 \cdot 10^{-5}$
1.250	0.625	$7.1 \cdot 10^{-5}$	$7.8 \cdot 10^{-5}$	$8.5 \cdot 10^{-5}$
1.300	0.665	$3.1 \cdot 10^{-4}$	$3.3 \cdot 10^{-4}$	$3.6 \cdot 10^{-4}$
1.379	0.730	0.0013	0.0014	0.0016
1.437	0.778	0.0028	0.0030	0.0033
1.507	0.837	0.0057	0.0062	0.0067
1.562	0.884	0.0092	0.010	0.011

Table 4. $\sigma(pp \to nn\pi^+\pi^+)$, nb

		ξ				
p_p, GeV/c	E, GeV	1.0	0.5	0	-0.5	-1.0
1.250	0.625	0.65	0.37	0.17	0.05	$0.5 \cdot 10^{-4}$
1.300	0.665	1.41	0.85	0.40	0.11	$1.2 \cdot 10^{-3}$
1.379	0.730	10.9	6.3	2.8	0.78	$8.5 \cdot 10^{-3}$
1.437	0.778	26.8	15.4	7.1	2.0	0.021
1.507	0.837	60.9	29.9	16.1	4.5	0.048
1.562	0.884	102	58.6	28.9	7.5	0.085

one-pion and two-pion diagrams) and include into consideration "non-chiral" mechanisms, e.g. formation of different resonances (Δ_{33}, ρ, σ) in the intermediate state. Taking all that into account may completely change our conclusion about high sensitivity of this reaction cross section to ξ at higher energies. And finally, note that for this reaction cross section it is impossible to carry out a comparison to experimental data (even similar to that we made for two other reactions): the lowest energy E_p, for which the cross section is measured, is equal to 1.078 GeV, and to apply the model of this paper to such an energy (almost 0.5 GeV above threshold) is absolutely senseless.

6 Conclusion

The present study is of a preliminary character. But it shows that, first, to shed light on many questions of nonlinear pion-nucleon dynamics it is necessary to have more complete experimental data on the $N(p,2\pi)2N$ reactions not far from threshold, and second, the model used can be a starting point for further detailed theoretical investigations of those reactions at the relevant energy region. Such investigations have to include consideration of different isospin channels and of "non-chiral" mechanisms, e.g. alternative ways of the $N^*(1440)$ excitation, excitation of resonances (Δ_{33}, ρ, σ etc.). In particular, it would be useful to reinvestigate the description of $\pi\pi$ interaction. In this connection we mention an attempt of that kind made in [3] while considering $\pi\pi$ interaction in terms of the linear σ model to describe some features of $N(\pi,2\pi)N$ reactions. A similar (but not identical!) mechanism could appear to be a reason of relatively high values of the $pp \to pn\pi^+\pi^0$ cross section [4], which are one order of magnitude bigger than the $pp \to nn\pi^+\pi^+$ cross sections, in spite of the fact that for both of them the $N^*(1440)$ mechanism does not contribute. In this connection it should be noted that even very close to threshold (where there are no experimental data so far) the cross section for the $pp \to pn\pi^+\pi^0$ channel is also expected to be bigger than the $pp \to nn\pi^+\pi^+$ cross section, because the number of diagrams (of the Fig. 2 type) contributing for the former is essentially bigger than for the latter. Certainly, a conclusive explanation of such a difference in those cross section values is to be given as a result of a detailed study of the $pp \to pn\pi^+\pi^0$ reaction together with the $pp \to nn\pi^+\pi^+$ and $pp \to d\pi^+\pi^0$ channels.

Also, it is interesting to investigate the $(p, 2\pi)$ reactions on nuclei close to threshold both theoretically and experimentally. There is nothing made so far in this direction except some scarce experimental data [16,17]. In this connection proposals on the experimental study of the energy dependence of $(p, 2\pi)$ reaction cross sections near threshold suggested at IUCF–TRIUMF [17] and at Moscow Meson Factory [22] seem to be worthy of attention and support.

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